Abstract

This paper examines the relevance of one of the most important textbook of endogenous growth models. The AK model –chosen as a representative of an endogenous growth model– is applied and tested using Chilean annual data (1960-1998). Chile’s case provides an interesting example of an economy showing a respectable growth rate that could either be characterized as endogenous or as neoclassical (transitional or steady state). The analysis revealed two main insights. First, when analyzing long-term growth, the time series should be purged from short-run fluctuations. This can be achieved utilizing a filter. Second, in principle, the differences between the AK model and the neoclassical growth model might not be so significant, making it difficult to discriminate between them from both a theoretical and an empirical point of view.
I. Introduction

Basically all (development) economists emphasize the fostering of savings and investment as crucial to successful economic development and economic growth.

Although the role of savings and investment –not neglecting the importance of an undistorted incentive system, good institutions, the protection of property and human rights, social and political stability...– is widely recognized, the ‘key’ question certainly remains whether a policy, which pushes savings and investment, will lead to a one-time increase of the rate of economic growth or, whether savings and investment will have a permanent (long-term) impact on the rate of growth. This issue is directly linked to the relevance/irrelevance of the neoclassical growth theory versus the endogenous growth theory.

In this study the author’s model of choice within the endogenous growth framework is the AK model. This model takes output to be a linear function of the accumulable factor capital (in a broad sense). Broad capital encompasses physical and human capital and is assumed to have constant returns to scale. In the AK model approach capital is the only determinant of the long-term growth rate.

The cornerstones of the neoclassical and endogenous growth models will be summarized in Section II.

In order to bring more light onto the theoretical debate outlined above, the main empirical findings of Jones (1995a) who analyzed the US and 14 OECD economies, will be highlighted in Section III.

An empirical test of an endogenous growth model, run by the author, will follow in Section IV. The AK model –taken as a representative of an endogenous growth model– will be applied and tested using Chilean data. The Chilean economy showed quite a successful growth path that could either be characterized as endogenous or as neoclassical (transitional or steady state). Since Chile is often treated as a ‘success story’ (especially since 1985), it is important to get some hints on whether Chile can be considered a country that was capable and able to generate endogenous (long-run) growth in the period under scrutiny. For this purpose the Chilean growth experience in the period of 1960-1998 will be analyzed in very general terms (Sections 4.1 and 4.2). The AK model will be applied to two sets (unfiltered and filtered) of Chilean data (Sections 4.3 and 4.4). Statistical methods will be presented to gain insights into the duration of the impact of economic policies, which produce an upward shift of savings and investment. This procedure has the purpose to determine whether economic policy has (had) a permanent or only transitory impact on economic growth.

In Section V conclusions concerning the relevance of the endogenous growth model, and more specifically the AK model, will be drawn. It will be pointed out that the conclusions depend largely on the definition of the long-run. Finally a future line of research will be referred to. This ‘in between approach’ puts emphasis on dynamic modelling, which allows to determine the duration of impact of certain policy variables in terms of years, months etc.
II. Transitory Versus Permanent Effects of Savings and Investment

The effects of a rise in savings and investment ratios on growth can be very different, depending on the growth theories one chooses to look at. According to the neoclassical growth model a positive shift of savings and investment rates will only have a temporary effect on the rate of growth, whereas according to the endogenous growth theory (in its AK version) it will have a long-run impact on the rate of growth.

2.1 The neoclassical growth model

According to the neoclassical growth theory a policy that promotes savings and investment will lead to an increase of output (level effect), but only to a short-run increase in the rate of growth (growth effect). The time-limited increase in the rate of growth is due to diminishing returns of the input factor: capital. Therefore, a rise in the savings rate which translates into investment will raise the level of per capita income and its growth rate only temporarily, up until the point at which the available savings is only sufficient to cover depreciation and growth in the labor force. Capital per worker stops increasing, although savings and investment continue to take place. This means that growth in per capita income would also stop, if there were no technological change. Viewed another way, during transition (most probably in the short and medium run) growth in per capita income is possible even without technological change, but in the long-run the growth in per capita income is just equal to the rate of technological change, and is entirely dependent on technological change which is exogenous (Solow, 1956; Swan, 1956; Mankiw, Romer and Weil, 1992; Barro and Sala-i-Martin, 1995).

2.2 The endogenous and the semi-endogenous growth model

Endogenous growth theory can explain long-run increases in output growth rates because of several phenomena, which are interdependent (Rebelo, 1991, 1998). In a first class of models, endogenous technical progress via knowledge accumulation and R&D makes the long-run growth permanent. Innovation, imitation and adaptation are driven by the profit-maximizing behavior of firms. Even though externalities might be connected with those activities, the costs of innovating or imitating new products are covered by temporary profits earned by mark-up pricing under imperfect competition, an idea already propagated by Schumpeter (Judd, 1985; Romer, 1990, 1994; Aghion and Howitt, 1992; Grossman and Helpman, 1991; Young, 1993). In this framework subsidies to R&D could enhance knowledge accumulation, which, in turn, raises the long-run growth rate permanently. A speed-up in technical progress could be caused by capital goods imports, increased transfer of technology, higher foreign direct investment and more incentives to imitate and innovate.

In a second class of models, externalities in the process of factor accumulation lead to permanent growth. Positive externalities linked to investment lead to
constant returns to scale in capital accumulation (Romer, 1986, 1987). While Romer refers to positive externalities of physical investment and investment-related knowledge, Lucas points to the positive externalities of human capital accumulation. (Romer, 1986; Lucas, 1988; Azariadis and Drazen, 1990; Murphy, Shleifer and Vishny, 1989). A mathematical simplification of these models is the AK model. It starts from the assumption of constant returns to scale of a factor ‘capital’ which comprises physical and human capital and the ‘unimportance’ of nonreproducible factors, such as land. ‘Capital’ accumulation becomes thus a profitable long-run business (Jones and Manuelli, 1990; Rebelo, 1991; Jones, 1995b). In these models externalities lead to the result that one-time improvements in efficiency can permanently increase the rate of economic growth. The new models of endogenous growth highlight also the role of international externalities for growth processes of developed and developing countries.

Semi-endogenous growth theory combines aspects of both the neoclassical growth model and the R&D-based endogenous approaches to growth (Jones, 1995b, 1998; Arnold, 1999; Segerstrom, 2000). The mechanism of growth in semi-endogenous growth models is profit-oriented innovation. However, due to a non-linear relationship between knowledge and innovation the long-run growth rate is determined by population growth (human capital growth). The transition period towards steady state growth is much longer than in the neoclassical model. Whether economic policy has an impact on long-run growth depends on the specification of the semi-endogenous growth model.

2.3 The mediation approach

It has to be pointed out, however, that the growth rate in the neoclassical model during transition is in fact an endogenous function of underlying parameters, and actual economies spend most or all of the time in a transitional state. In contrast, the endogenous growth model shows that economic policies which enhance the rate of saving and investment have an impact on the long-run rate of growth.

This is meant to say that –after all– the differences between the neoclassical model and the endogenous model are not that big, if the transitional state is the rule or lasts for years. Under comparable time intervals, capital accumulation, the rate of depreciation, the rate of population growth/labor force growth and –of course– the rate of technological progress have a similar impact on the growth rate of output.

The only difference would be that in the endogenous growth model the rate of technological advance is explained by profit-maximizing firm decisions to imitate and/or to innovate. Besides, the existence of externalities (spillovers) makes the accumulation of physical and human capital and knowledge more attractive, thus enabling higher rates of output growth if certain positive conditions are fulfilled. Therefore, the issue of the impact of ‘good’ economic policies on economic growth remains a matter for empirical testing.
III. Jones’ Test of the AK Model: Findings for the U.S.A. and 14 OECD Countries

In order to clarify the role of good economic policies empirical analyses become necessary. Jones (1995a) studied the relevance of endogenous growth for the U.S.A. and 14 OECD countries. Even though his results are clear, they remain primarily country specific. However, Jones’ findings may even be distorted and misleading because the data have not been purged from short-run fluctuations and because structural changes have not been taken into account. This point will be picked up in Sections 4.2 and 4.4.

Jones (1995a) was the first who did time series tests of endogenous growth models. He examined the growth rate of the US economy (1880-1987) and of 14 OECD countries (1900-1987) by applying time series tests. He looked at the time series properties of the per capita GDP growth in the United States and concluded from its constant mean and its stationarity (in the statistical sense) that ‘‘either nothing in the U.S. experience since 1880 has had a large, persistent effect on the growth rate, or whatever persistent effects have occurred have miraculously been offsetting’’. The same applied to the fourteen OECD countries4 when looking at the ADF-test5, which proved the growth rates to be stationary. These results call into question the implicit prediction of many endogenous growth models for the countries under investigation, that growth rates should exhibit large permanent increases (Jones, 1995a).

However, if one examined the period of 1950-1988, the picture is mixed. One would realize a positive mean shift after World War II. The countries with significant mean shifts are Australia, Austria, Germany, Italy, Japan and the United Kingdom. With the exception of Australia, these were all countries that were severely affected by the war and where the recovery in the ensuing decades was tremendous due to the Marshall Plan which facilitated the inflow of capital.

To sum up: Endogenous growth could not be detected for the US and the OECD economies over periods of about 100 years. The AK model had to be rejected (with the exception of the economies most destroyed by the war) when analysing the period of 1950-1988.

IV. An Analysis of Chilean Growth: Does the AK Model Apply to the Chilean Economy?

4.1 The Chilean growth experience and growth prospects

The Chilean growth experience is well documented in the book “Análisis empírico del crecimiento en Chile” edited by Morandé and Vergara (1997). Some stylized facts and main findings shall be gathered in the following paragraphs.

Lefort (1997) points out that Chile was the only economy in Latin America that increased its growth rate in the period of 1975-1990 compared to the period

According to Rojas, López and Jiménez (1997) GDP growth in Chile averaged 3.9% in the period of 1961 to 1996. Capital growth contributed ~ 40%, labor growth contributed ~ 60% to GDP growth and TFP growth contributed close to nothing or even a bit negatively to output growth. In the period of 1991-96, GDP growth averaged 7.4%. In that period capital growth caused 30%, labor growth caused 39% and TFP growth caused 31% of this tremendous output growth (Vergara, 1997). The question of course is whether this impressing growth of the early nineties can go on forever.

Camhi, Engel and Micco (1997) observed a strong increase in TFP growth in the Chilean manufacturing sector in the exportable branch from 1991 to 1996. This upswing in TFP growth was paralleled by a 30%-fall (appreciation) in the real exchange rate (Vergara, 1997). However, it has also to be questioned whether such a continous real appreciation is sustainable in the long-run (forcing producers of tradeables to become more and more productive). At least the recessionary experience of 1998/99 seems to contradict the view of those long-lasting appreciations not being harmful to the real side of the economy.

De Gregorio (1997) predicts potential growth in Chile to be in the range of 6.5% to 7% in the long term, depending on some optimistic assumptions on the rate of investment (more than 20%) and the rate of productivity growth (3%).

Roldós (1997) comes to very similar projections concerning Chile’s growth potential and points to the importance of the quality of the input factors for enhancing growth. The percentages of (imported) machinery and equipment (standing for capital goods) and learning-by-doing (standing for labor) are considered to be growth promoting factors.

4.2 Overview of Chilean growth in the period of 1960-1998

In this section a quick look shall be taken at the data such as computed by the author. The dataset underlying the statistical analysis comprises GDP, capital stock and occupation data for the period of 1960-1998.6 First, the line graphs of Y, K, L, LNY, LNK and LNL in Figures 1 and 2 will be looked at (see Appendix 1).

Y stands for GDP in real terms measured in millions of 1986 pesos. The variable K indicates the capital stock in real terms (millions of 1986 pesos). It stands for gross capital formation (in analogy to the Jones’ data) and is composed of change in stock and gross fixed capital formation. And finally L is an indicator of the number of occupied persons, measured in thousands of persons. The data with the prefix LN are logarithms of Y, K and L. Both data sets reveal a decline in the 1973-75 period (first recession). In the year 1982 the country was hit by a
second recession, which was reinforced by the debt crisis. Starting in 1985 the economy recovered steadily. A third recession occurred in 1999 which, however, is not covered by the data. Both the Y-K-L-series and the LNY-LNK-LNL-series were non-stationary, i.e. they exhibited clear upward and downward trends.

Second, the line graphs of the growth rates WY, WK and WL in Figures 3 and 4 (see Appendix 1) show a tremendous amount of oscillation, but no increasing or decreasing trend. They seem to fluctuate around a constant mean. Stationarity was confirmed by the Phillips-Perron test, but not under Kapetanios’ critical values. The growth rates were created in the following way:

\[ WY_t = \text{LNY}_t - \text{LNY}_{t-1} \]
\[ WK_t = \text{LNK}_t - \text{LNK}_{t-1} \]
\[ WL_t = \text{LNL}_t - \text{LNL}_{t-1} \]

Third, the growth rates relevant for the AK model are purged from short-run fluctuations so that a possible trend can become visible, generating two sets of variables: HPWY, HPWK and WYD and WKD. (Appendix 1: Figures 5 and 6). One of the methods used for purging the data is the Hodrick-Prescott (HP) filter. HP, therefore, is the abbreviation for ‘filtered’ according to this method. The HP filter is a very strong filter that can produce an extreme smoothing of the series. Another method for smoothing the series WY and WK is the double smoothing method (abbreviated by ‘D’) that allows to create the series: WYD and WKD.\(^8\) WYD’s smoothing parameter is 0.09\(^9\) and that of WKD’s is 0.04\(^10\). The series WYD and WKD are less smoothed than HPWY and HPWK. Obviously the HP-filter works in a different way and can involve an increasing smoothing parameter. Whereas the double smoothing technique\(^11\) relies on a constant smoothing parameter (\(\alpha\)) to be estimated, the HP-smoothing technique tries to minimize the variance of the series around the smoothed series (\(s_t\)), subject to a penalty that constrains the second difference of \(s_t\). The HP-smoothing method is widely used among macroeconomists to obtain a smooth estimate of the long-term trend component of a series.\(^12\)

Fourth, TFP growth (WTFP) is computed as a residual (see Appendix 1: Figure 4). For this purpose the values for the output elasticities are taken from Coeymans (1999a) and Coeymans (1999b). Coeymans estimates the output elasticity of capital to be 0.35 and the output elasticity of labor (employment) to be 0.65. Constant returns to scale are assumed and ‘confirmed’ by a test on this restriction (Coeymans, 1999b).

\[ \text{WTFP}_t = WY_t - 0.35 \, WK_t - 0.65 \, WL_t \]

Fifth, some time series properties, such as stationarity/non-stationarity of the series and tests on it are to be discussed. The conventional unit root tests can be
looked up in Darnell (1994), Harvey (1995), Hendry (1995), Lütkepohl (1993) and many other statistical books. It is important to note that the results of the standard unit root tests (such as the Phillips-Perron test or the augmented Dickey-Fuller test) are not robust in the presence of structural and trend changes. Therefore, new critical values have to be computed. These critical values are to be gained by simulation and are higher (in absolute terms) than the typical MacKinnon critical t-values. They increase with the number of structural breaks and are dependent on whether structural breaks have affected the slope parameter, the intercept or both (Kapetanios, 1999). Therefore, the statistical properties of the series, which become important in the analysis of the AK model, are all checked by a conventional and a non-standard test of stationarity (taking structural change into account). They are subject to a cointegration test whenever possible. The results are summarized in Appendix 2: Tables 1-3.

4.3 Test of the AK model using non-filtered data

Let us now look at the AK model such as outlined by Jones (1995a) and its implications concerning long-run growth (see equation (1)).

Households maximize their utility by choosing $i^k$, $i^h$

$$\max \int_0^\infty e^{-\rho t} u(c_t) dt$$

subject to:

$$c_t = (1 - i^k - i^h) y_t$$
$$y_t = A k_t^\alpha h_t^{1-\alpha}$$
$$\dot{k}_t = i^k y_t - \delta k_t$$
$$\dot{h}_t = i^h y_t - \delta h_t$$

where:

$u( )$ = CRRA utility function with intertemporal elasticity of substitution $\sigma$
$c$ = consumption
$\delta$ = rate of depreciation (assumed to be the same for both types of capital)
$\rho$ = rate of time preference
$i^k$, $i^h$ = investment ratios of physical and human capital, respectively

Constant returns to the accumulable factors are assumed, which will generate endogenous growth.

When solving equation (1), one can prove that the ratio $h/k$ is constant and equal to $(1-\alpha)/\alpha$. Since adjustment costs are assumed to be non-existent, the model
...instantaneously adjusts the initial amounts of k and h so that this ratio is always achieved. Therefore, the two types of capital can be said to develop in a parallel way. This leads one to rewrite the production function in terms of a simplified production technology (see equation (2)):

\[ Y_t = \tilde{A} K_t^\delta \]  

with:

- \( Y \) = GDP in real terms
- \( \tilde{A} = A (h/k)^1-\alpha \)
- \( K \) = physical and human capital, represented by physical capital k
- \( t = \) time/years (1960-1998).

Production in this model exhibits constant returns to the accumulable factor: K, which will generate endogenous growth. The equilibrium growth conditions imply that physical and human capital grow at the same rate, such that the development of physical capital can be taken as synonymous with the development of human capital. Since reliable data on the development of human capital are often lacking, they are replaced by data on physical capital. k is thus treated as representative of K.

To analyze the steady state relationship between the growth rate (WY) and the investment rate (WK), one has to take logs and differentiate (2) to get to (3):

\[ W_k = -\delta + \tilde{A} i_k = W_h = W_K = W_Y \]  

with:

- \( W_k \) = rate of growth of the physical capital stock
- \( W_h \) = rate of growth of human capital
- \( W_K \) = rate of growth of the total capital stock
- \( W_Y \) = growth rate of real GDP
- \( \delta \) = rate of depreciation, assumed to be the same for physical and human capital
- \( \tilde{A} = A (h/k)^{1-\alpha} \) = production technology; productivity parameter
- \( i_k \) = investment rate for physical capital

Jones (1995a) concludes from equation (3) that in the AK model the dynamics of the growth rate should be similar to the dynamics of the investment rate (i.e., growth rate of the physical capital stock).
Following this line of thought the AK-model will be tested. Endogenous growth—according to the AK-model—requires that an increase of the growth of the physical capital stock ($W_k = W_{K,13}$) is paralleled by an increase in output growth ($W_Y$) over the long run. In case the AK-model is rejected, validity of the neoclassical model would be taken as granted (Jones, 1995a).

By doing this, one will notice that series $W_Y$ and $W_K$ look quite similar. However, the robust, non-standard unit root test revealed the non-stationarity of $W_Y$ and the stationarity of $W_K$; i.e., different time-series properties (compare Figure 3 in Appendix 1 and Table 2 in Appendix 2). These results do not favor the AK model.

4.4 Test of the AK model using smoothed/filtered growth rates

Overall, structural changes in Chile (in 1973?, in 1975?, in 1982?, and may be in many more years) make it hard to interpret and understand the development of the Chilean time series.

However, provided those structural changes that characterize Chile’s economic history in the period of 1960-98, one should not treat the structural break years as outliers or missing values.

It is interesting to note that—utilizing a conventional unit-root test14—the growth rate of the GDP in the period of 1960 to 1974 ($W_{Y,60-74}$) was tested non-stationary and that of the period of 1975 to 1998 ($W_{Y,75-98}$) was tested to be stationary. In accordance to that result, the growth rate of capital in the period 1960 to 1974 ($W_{K,75-98}$) was tested non-stationary and that of the period of 1975 to 1998 ($W_{K,75-98}$) was tested stationary (Appendix 2: Table 1).

Under the heroic assumption of only one structural break in 1975, this would point to a co-movement of the series and would favor the AK model. Given this result, the finding of section 4.3 does not seem to be robust.

The finding of non-robust results leads one to wonder how the long-run behavior of the time series would look like. Structural changes and manyfold short-run fluctuations seem to cloud the long-run relationship between $W_Y$ and $W_K$, which is to be tested. Following Coeymans (1999a, 1999b) one should be aware and take care of the tremendous short-run fluctuations that are mainly due to fluctuations in capacity utilization, to changes in the real exchange rate and in the terms of trade (see also Easterly et al., 1993).

Therefore, it was decided to purge the annual growth rates from those fluctuations, by applying two smoothing methods. First, the Hodrick-Prescott filter is applied and the series: $HP_{WY}$ and $HP_{WK}$ are generated. These new series are depicted in Figure 5 in Appendix 1 and certainly show some downward and upward movements. These movements point to non-stationarity in the series. Non-stationarity is ‘confirmed’ by applying a robust unit-root test (see Table 2 in Appendix 2). Second, the double smoothing method is utilized and the series $W_{YD}$ and $W_{KD}$ are created. They are shown in Figure 6 in Appendix 1 and also
feature some downward and upward movements, but are not smoothed in such an extreme way as HPWY and HPWK. The non-standard unit root test reveals the non-stationarity of the series (Table 2 in Appendix 2).

These results take us one step further. Given the fact that these series are non-stationary (integrated of order I(1)), it can now be tested whether they are cointegrated; i.e., whether there exists a long-run equilibrium between HPWY and HPWK (or WYD and WKD) as the AK model would suggest.15

If the question of a cointegrating relationship between HPWY and HPWK (or WYD and WKD) is answered with ‘yes’, then we would have a hint that increasing growth rates are sustainable in the ‘long-run’, provided that a period of approximately forty years can be called the ‘long-run’. If the answer is ‘no’, then we would have to conclude that rather the neoclassical growth model applies where no ‘long-run’ relationship between the growth of the capital stock and the output growth exists.

Therefore, the endogenous growth model16 is applied to the smoothed series HPWY and HPWK (plus WYD and WKD), which is in contrast to Jones (1995a), but certainly makes much more sense from an economic point of view (see Figures 5 and 6 in Appendix 1 for the interplay between HPWY and HPWK and alternatively WYD and WKD).

Cointegration between HPWY and HPWK (plus WYD and WKD) was then tested by means of the Johansen cointegration test (Johansen, 1987). The test detected two cointegrating vectors, that is cointegration as far as HPWY and HPWK are concerned. The computed cointegrating vector makes sense from an economic point of view. The computed output elasticity of HPWK is 0.3517 and significant for $\alpha = 1\%$. One cointegrating vector is found concerning WYD and WKD indicating long-run equilibrium between both series.

It should be pointed out that the finding of non-stationarity of the smoothed series and that of cointegration remain robust if two other smoothing techniques (such as the single smoothing and the Holt-Winters smoothing method) are applied.

The result of cointegration is in line with Jones’ (1995a) observations on some OECD countries after WWII (1945-1987) who experienced increasing growth rates due to the process of reconstruction. However, over the whole period of 1900-1987, Jones did not observe increasing growth for the very same countries! It has to be kept in mind –of course– that Jones analyzed WY and WK, series which contain all the (misleading!) short-run fluctuations.

V. Conclusions

Since short-run fluctuations tend to conceal important medium to long-run trends, the author decided –in contrast to Jones (1995a)– to purge the annual growth rates WY and WK from those swings, thus creating HPWY and HPWK, and alternatively WYD and WKD.
As far as Chile is concerned a parallel upward movement between the smoothed growth rate of Y (HPWY/WYD) and the smoothed growth rate of K (HPWK/WKD) could be detected for the period under consideration (1960-1998). It could even be shown that both series were cointegrated; i.e., in long-run equilibrium (for 39 years).

Jones (1995a), in contrast, did not encounter a parallel upward movement between WY and WK (unsmoothed series) for the period of 1950-1988 for the U.S.A. and for the majority of the 14 OECD countries. This led him to reject the endogenous growth model (AK model). However, when looking at shorter time periods (1945-1987) Jones could also detect an upward co-movement of WY and WK for some 'war-destructed' OECD countries, such as Germany, Austria, Italy, Japan and UK.

Arnold (1997, 1999), another critic of the endogenous growth model, does not reject the endogenous growth model per se, but makes proposals to modify some of the unrealistic assumptions of endogenous growth theory in order to make the theory fit the facts. This line of research seems to be quite promising.

So, where do we stand? Do we have to assume a neoclassical or an endogenous growth for Chile? The answer depends on the definition of the long-run. If we consider 39 years as long run, then we would have had endogenous growth in Chile. If we consider the 1960-1998 period too short to be classified as long-run, we could say that we are in the stage of transition to a new steady state and that the neoclassical model applies.

Concerning economic policy, one might be induced to say that policy matters for a fairly long period, too long not to worry about its being good or bad! However, in order to make more concrete statements on the impact of certain policies over time one should revert to dynamic macroeconometric models, such as distributed lag models. This should be a line of research to be followed in the future.
### APPENDIX 1

#### TABLE 1

THE ORIGINAL DATA: Y, K, L

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<td>1982</td>
<td>2840122</td>
<td>537389.4</td>
<td>3039.436</td>
</tr>
<tr>
<td>1983</td>
<td>2819928</td>
<td>457435.2</td>
<td>3120.543</td>
</tr>
<tr>
<td>1984</td>
<td>2998736</td>
<td>498504.5</td>
<td>3336.086</td>
</tr>
<tr>
<td>1985</td>
<td>3072177</td>
<td>572188.0</td>
<td>3524.197</td>
</tr>
<tr>
<td>1986</td>
<td>3246107</td>
<td>586023.0</td>
<td>3709.040</td>
</tr>
<tr>
<td>1987</td>
<td>3644681</td>
<td>713263.0</td>
<td>3867.340</td>
</tr>
<tr>
<td>1988</td>
<td>3911154</td>
<td>814209.0</td>
<td>4059.560</td>
</tr>
<tr>
<td>1989</td>
<td>4324181</td>
<td>1058456.0</td>
<td>4293.700</td>
</tr>
<tr>
<td>1990</td>
<td>4484071</td>
<td>1083096.0</td>
<td>4398.750</td>
</tr>
<tr>
<td>1991</td>
<td>4841447</td>
<td>1083169.0</td>
<td>4421.680</td>
</tr>
<tr>
<td>1992</td>
<td>5435881</td>
<td>1343405.0</td>
<td>4643.070</td>
</tr>
<tr>
<td>1993</td>
<td>5815646</td>
<td>1584627.0</td>
<td>4894.980</td>
</tr>
<tr>
<td>1994</td>
<td>6147610</td>
<td>1682653.0</td>
<td>4969.900</td>
</tr>
<tr>
<td>1995</td>
<td>6600952</td>
<td>2078072.0</td>
<td>5018.040</td>
</tr>
<tr>
<td>1996</td>
<td>7305141</td>
<td>2263410.0</td>
<td>5141.500</td>
</tr>
<tr>
<td>1997</td>
<td>7858481</td>
<td>2526156.0</td>
<td>5194.900</td>
</tr>
<tr>
<td>1998</td>
<td>8126506</td>
<td>2579026.0</td>
<td>5257.239</td>
</tr>
</tbody>
</table>

Y = GDP in real terms (millions of 1986 pesos).
K = Capital stock in real terms (millions of 1986 pesos).
L = Employment (thousands of persons).

**Source:** Y, K: Boletín Mensual, various issues; Banco Central de Chile
L: Professor Coeymans’ data base; Universidad Católica de Chile, Santiago
FIGURE 1
THE DEVELOPMENT OF Y, K, L

Note:
Y, K, and L are non-stationary. The Johansen cointegration test indicated cointegration between them. However, the existence of a long-run equilibrium between those series is nothing exceptional. Besides, it does not allow conclusions on whether a neoclassical or an endogenous growth process prevails.
LNY, LNK and LNL are non-stationary. The Johansen cointegration test showed those series to be cointegrated as was to be expected (compare also Coeymans (1999b for the period of 1960-1997 and Rojas et al. (1997) for the period of 1960-1996). These findings, however, do not allow conclusions about whether one is confronted with a neoclassical or an endogenous growth model.
FIGURE 3

THE DEVELOPMENT OF WY AND WK
(VISUALIZATION OF THE AK MODEL)

Note:
WY and WK seem to be quite similar. However, a unit root test that takes structural change into account revealed WY to be non-stationary and WK to be stationary.
FIGURE 4
THE DEVELOPMENT OF WL AND WTFP

Note:
WL and WTFP are stationary. They fluctuate around a constant mean.
FIGURE 5
THE DEVELOPMENT OF HPWY AND HPWK
(VISUALIZATION OF THE AK MODEL)

Note:
HPWY and HPWK are growth rates that have been freed from their short-run fluctuations by means of the Hodrick-Prescott filter. They are non-stationary. According to the Johansen cointegration test, HPWY and HPWK are cointegrated, i.e. in the long-run they move together. Cointegration between HPWY and HPWK gives strong support to the AK model.
FIGURE 6
THE DEVELOPMENT OF WYD AND WKD
(VISUALIZATION OF THE AK MODEL)

Note:
WYD and WKD are non-stationary. They are cointegrated in the long-run. Cointegration between WYD and WKD gives strong support to the AK model.
APPENDIX 2

TABLE 1

THE TIME SERIES ARE SUBJECT TO A CONVENTIONAL UNIT ROOT TEST
(Phillips-Perron test\(^\text{20}\))

<table>
<thead>
<tr>
<th>series to be tested</th>
<th>test assumptions(^\text{21})</th>
<th>test result</th>
<th>PP test statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>1.02</td>
</tr>
<tr>
<td>K</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>1.06</td>
</tr>
<tr>
<td>L</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−0.98</td>
</tr>
<tr>
<td>LNY</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−1.02</td>
</tr>
<tr>
<td>LNK</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−0.89</td>
</tr>
<tr>
<td>LNL</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−1.42</td>
</tr>
<tr>
<td>WY</td>
<td>intercept</td>
<td>stationary</td>
<td>−3.95</td>
</tr>
<tr>
<td>WY6074(^*)</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−1.68</td>
</tr>
<tr>
<td>WY7598(^**)</td>
<td>intercept</td>
<td>stationary</td>
<td>−5.20</td>
</tr>
<tr>
<td>WK</td>
<td>intercept</td>
<td>stationary</td>
<td>−4.55</td>
</tr>
<tr>
<td>WK6074(^*)</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−1.95</td>
</tr>
<tr>
<td>WK7598(^**)</td>
<td>trend and intercept</td>
<td>stationary</td>
<td>−3.62</td>
</tr>
<tr>
<td>WL</td>
<td>intercept</td>
<td>stationary</td>
<td>−4.23</td>
</tr>
<tr>
<td>WTPF</td>
<td>intercept</td>
<td>stationary</td>
<td>−4.97</td>
</tr>
<tr>
<td>HPWY</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−2.02</td>
</tr>
<tr>
<td>HPWK</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−2.30</td>
</tr>
<tr>
<td>HPWL</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−0.91</td>
</tr>
<tr>
<td>HPWTPF</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>0.89</td>
</tr>
<tr>
<td>WYD</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−1.39</td>
</tr>
<tr>
<td>WKD</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>−1.16</td>
</tr>
</tbody>
</table>

\(^*:\) WY6074/WK6074 is the growth rate of real GDP/capital stock in the period of 1960-74.
\(^**:\) WY7598/WK7598 is the growth rate of real GDP/capital stock in the period of 1975-98.
TABLE 2  
THE SERIES RELEVANT FOR THE AK MODEL ARE SUBJECT TO A UNIT ROOT TEST  
THAT IS ROBUST IN PRESENCE OF STRUCTURAL CHANGE(S)

<table>
<thead>
<tr>
<th>series to be tested</th>
<th>test assumptions</th>
<th>test result</th>
<th>computed value and critical t-value (according to Kapetanios); $\alpha = 0.05$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
<tr>
<td>K</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
<tr>
<td>LNY</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
<tr>
<td>LNK</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
<tr>
<td>WY</td>
<td>intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-4.50) (critical value given 1 structural break) #</td>
</tr>
<tr>
<td>WK</td>
<td>intercept</td>
<td>stationary</td>
<td>computed value $\prec$ (-4.50) (critical value given 1 structural break) #</td>
</tr>
<tr>
<td>HPWY (smoothed WY)</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
<tr>
<td>HPWK (smoothed WK)</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
<tr>
<td>WYD (smoothed WY)</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
<tr>
<td>WKD (smoothed WK)</td>
<td>trend and intercept</td>
<td>non-stationary</td>
<td>computed value $\succ$ (-5.08) (critical value given 1 structural break)*</td>
</tr>
</tbody>
</table>

Note:
* : This is the critical value if only 1 structural break is observed. The critical t-values for 2 (3) structural breaks are \(-6.11\) (\(-7.01\)). It is assumed that the structural break affects trend and intercept.
# : This is the critical value if only 1 structural break is observed. The critical t-values for 2 (3) structural breaks are \(-5.01\) (\(-5.73\)). It is assumed that the structural break affects only the intercept.
### TABLE 3
RESULTS OF THE JOHANSEN COINTEGRATION TEST

<table>
<thead>
<tr>
<th>series tested</th>
<th>test assumptions</th>
<th>test result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y, K, L</td>
<td>linear deterministic trend</td>
<td>cointegration</td>
</tr>
<tr>
<td></td>
<td>(intercept)</td>
<td>(2 cointegrating eqs.)</td>
</tr>
<tr>
<td>LNY, LNK, LNL</td>
<td>linear deterministic trend</td>
<td>cointegration</td>
</tr>
<tr>
<td></td>
<td>(intercept)</td>
<td>(2 cointegrating eqs.)</td>
</tr>
<tr>
<td>WY, WK (test of the AK model)</td>
<td>cointegration test is not</td>
<td></td>
</tr>
<tr>
<td></td>
<td>indicated</td>
<td></td>
</tr>
<tr>
<td>HPWY, HPWK (test of the AK model</td>
<td>linear deterministic trend</td>
<td>cointegration</td>
</tr>
<tr>
<td>using smoothed series</td>
<td>(intercept and trend)</td>
<td>(2 cointegrating eqs.)</td>
</tr>
<tr>
<td>WYD, WKD (test of the AK model</td>
<td>linear deterministic trend</td>
<td>cointegration</td>
</tr>
<tr>
<td>using smoothed series</td>
<td>(intercept and trend)</td>
<td>(1 cointegrating eq.)</td>
</tr>
</tbody>
</table>

### Notes

1. Harrod (1939) and Domar (1946) propagated a first version (labor surplus version) of the AK model. P. Romer rediscovered and modernized its basic idea of constant returns to capital in 1986.
2. That is, being in the process of transition and moving towards a new steady state equilibrium.
3. Rebelo (1991) proved that perpetual growth can be consistent with the presence of capital goods produced with nonreproducible factors.
4. Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Sweden and the United Kingdom.
5. ADF-test = Augmented Dickey Fuller – test (a unit root test; test on non-stationarity/stationarity of time series).
6. The data, Y and K, have been taken from statistics of Chile’s Central Bank, and L has been provided by Prof. Coeymans; Universidad Católica de Chile, Santiago (see Appendix 1 for the data).
7. Non-stationarity makes the application of the wide-spread regression analysis at least questionable.
8. Two other smoothing methods, i.e. single smoothing and smoothing according to Holt-Winters, produced very similar series with the same time series properties.
9. 0.09 was estimated by EViews.
10. 0.04 was estimated by EViews.
11. The double smoothing technique expresses the smoothed series of $W_Y$ ($s_t$) as $s_t = \alpha^* W_Y + (1-\alpha^*) s_{t-1}$ and the double smoothed series ($d_{s_t}$) as $d_{s_t} = \alpha^* s_t + (1-\alpha^*) d_{s_{t-1}}$.
12. The working of smoothing techniques can be looked up in EViews 3 User’s Guide.
13. In EViews the names of series appear in capital letters, i.e. Wk the growth rate of physical capital is written as WK. This might be confusing, but in any case the growth rate of Wk (physical capital) and WK (human and physical capital) are the same under equilibrium growth conditions.
14. One should treat these results with caution, of course. First, the number of observations in the subsamples is quite small. Second, additional structural breaks in the sub-periods of 1960-74 and 1975-98 should be ruled out.
When performing the cointegration test, a requirement is non-stationarity of the series – a feature not fulfilled in the annual series. That is why Jones (1995) did not have to test for cointegration.

The AK model assumes some parallelism (i.e. cointegration in the statistical sense) between WY and WK.

The same output elasticity was calculated by Coeymans using non-filtered data.

It is advisable to take smoothed series in order to avoid biased conclusions which are caused by short-run fluctuations.

Jones would probably agree with that.

The Phillips-Perron test (unit root test) is a test on the stationarity/non-stationarity of the series. The Phillips-Perron test was applied to all series listed in the table. Another possible conventional unit-root test (test on stationarity/non-stationarity) is the augmented Dickey-Fuller test (ADF test).

The test assumptions follow from the line graphs.

The test assumptions follow from the line graphs.

The cointegration test requires non-stationarity of the series!

Assumptions underlying the series.

References


